Worcester County Mathematics League

WOCOMAL Varsity Meet #2

Coaches' Booklet

December 7, 2005

Round 1:   Fractions, Decimals, and Percents

ALL ANSWERS MUST BE INTEGERS.

1.   What digit is in the 2005th place after the decimal point in the decimal expansion
        of the fraction $\dfrac{1}{81}$?

2.   A copying machine can make copies that are 80%, 100% and 150% as large as the
        original. By making copies of copies, what is the smallest number of times you
        must use the machine if you want a copy of an original that is 324% as large as
        the original?

3.   If the surface area of one cube is 44% larger than the surface area of a second
        cube, what is the **closest whole percent** by which the volume of the first cube
        exceeds the volume of the second cube?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Auburn, Leicester, Notre Dame Academy

Round 2:   Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   A line through the points (m, -9) and (7, m) has slope m. What is the value of m?

2.   The difference between two numbers is 4. A quotient of the two numbers is -6.
         What is the largest possible value for either number?

3.   Suppose that $a + b = 3$ and $a^2 + b^2 = 7$. Then what is the value of $a^4 + b^4$?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Bartlett, Quaboag, and Bromfield

Round 3:   Parallel Lines and Polygons

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   The equal sides of an isosceles triangle have lengths of $(5x - 30)$ and $(3x + 10)$.
        What is the greatest possible integer value of the third side?

2.
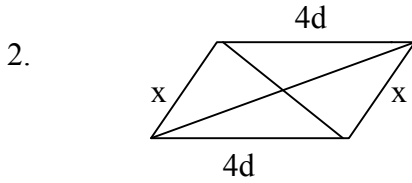
        Two sides of a parallelogram (as marked above on the figure) have lengths 4d.
        The two diagonals are of lengths 4d and 6d. How long are the other sides x of
        the parallelogram? (**The answer should be written in terms of "d".**)

3.
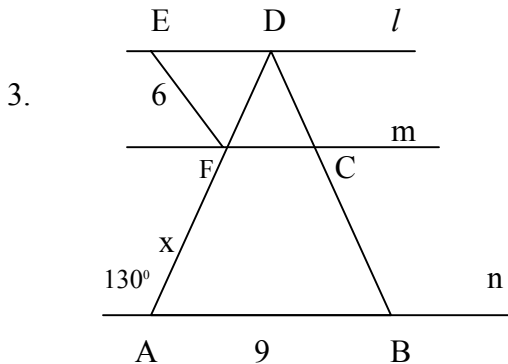
In the figure at the left the lines l,
m and n are all parallel. FC = 4 and <DEF = 50º.
What is the value of x = AF?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Westborough, St. John's, Tantasqua

Round 4:   Sequences and Series (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Given that the fourth term of a geometric sequence is 108 and the seventh term is 2916, what is the first term of the sequence?

2.   The difference between the square of the arithmetic mean of two numbers and the square of their geometric mean is 49. Find the difference between the arithmetic mean of their squares and the square of their arithmetic mean.

3.   The first three terms of an arithmetic sequence, in order, are $2x + 4$, $5x – 4$, and $3x + 4$. What is the sum of the first ten terms of the sequence?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Hudson and Bartlett

Round 5:   **Matrices and Systems of Equations (NO CALCULATORS)**

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   For the system: $x - 2y + z = 16$, $2x - y - z = 14$, $3x + 5y - 4z = -10$, Find the value of $(x + y + z)$.

2.   The determinant of the matrix $\begin{pmatrix} x+7 & x+9 & -2 \\ 8 & x+4 & -4 \\ 2 & 3 & -1 \end{pmatrix}$ is zero for two values of x.

What is the sum of these two values?

3.   Solve $\begin{pmatrix} 1 & -2 \\ 4 & 3 \end{pmatrix} + 2\mathbf{X} = \begin{pmatrix} 2 & 1 & -2 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 1 & 4 \\ -3 & 1 \end{pmatrix}$ for **X.**

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Clinton, Worcester Academy, and Algonquin

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET.   (2 points each)

1.   Drain pipe A drains a pool that is $\dfrac{2}{5}$ full in 3 hours. Drain pipe B drains the same pool when it is

$\dfrac{1}{4}$ full in 1.25 hours. How long would it take both drain pipes, working together, to drain a pool

that is $\dfrac{2}{3}$ full? **Write your answer in terms of hours.**

2.   Three different nonzero numbers a,b, and c, are chosen so that
$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b}$$

What is the common value of these three fractions?

3.   Three positive whole numbers x,y,z form a geometric sequence with a sum of 42. If y is

multiplied by $\dfrac{5}{4}$, the numbers x, $\dfrac{5y}{4}$, z now form an arithmetic sequence. What is the largest

possible value of x?

4.   The interior angles of a convex polygon of 9 sides are in arithmetic progression. If the smallest interior angle is 112^0, then what is the largest interior angle degree measure?

5.   A strictly increasing geometric sequence starts with 5. An arithmetic sequence also starts with 5 and happens to have its 4th and 16th terms equal to the geometric sequence's 3rd and 5th terms, respectively. What is the sum of the first 10 terms of the arithmetic sequence?

6.   Find the number of integers n such that $\dfrac{5n+26}{2n+3}$ is an integer.

7.   One root of $mx^2 - 10x + 3 = 0$ is two-thirds of the other root. What is the sum of the roots?

8   If the system of equations $\begin{cases} 2x - 3y = -4 \\ 3x - y = 1 \\ x - ky = 5 \end{cases}$ has a solution, determine the value of k.

9.   After ten tests, Bryan's average test grade was 84. His teacher decided to drop each student's lowest and highest test grade. This brought Bryan's average up to 85. If his lowest test score (of the original 10 tests) was 24 points lower than his highest test, what was his highest test grade?

Bartlett, Bromfield, Leicester,Worcester Academy, St. John's

Round 1: Fractions, Decimals, and Percents

1.   **(1 pt.)**   6

2.   **(2 pts.)**   6

3.   **(3 pts.)**   73

Round 2: Algebra 1-open

1.   **(1 pt.)**   3

2.   **(2 pts.)** $\dfrac{24}{7}$

3.   **(3 pts.)**   47

Round 3: Parallel Lines & Polygons

1.   **(1 pt.)**   139

2.   **(2 pts.)** $\mathbf{d}\sqrt{10}$

3.   **(3 pts.)**   7.5

Round 4: Sequences and Series

1.   **(1 pt.)**   4

2.   **(2 pts.)**   49

3.   **(3 pts.)**   176

Round 5: Matrices

1.   **(1 pt.)**   4

2.   **(2 pts.)**   5

3.   **(3 pts.)** $\begin{pmatrix} 6 & 4 \\ -6 & 2 \end{pmatrix}$

TEAM ROUND (2 pts. Each)

1.   2

2.   -1

3.   24

4.   168

5.   275

6.   4

7. $\dfrac{5}{4}$

8.   -2

9.   92

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET.   (2 points each)

1.   _____

2.   _____

3.   _____

4.   _____

5.   _____

6.   _____

7.   _____

8.   _____

9.   _____

TEAM ROUND　　　　　　**School: _____**

　　　　　　　　　　　　　　　Team #:_____

Team Members:

1.　　_____

2.　　_____

3.　　_____

4.　　_____

5.　　_____